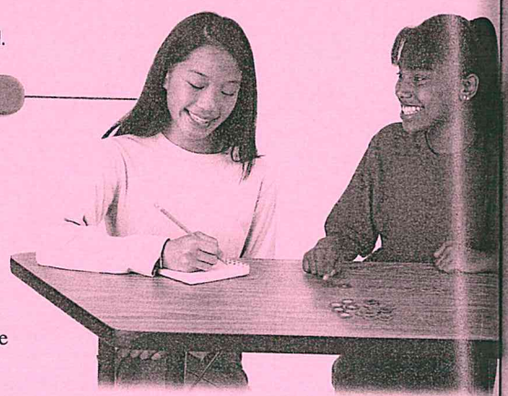
6.6 Using Counters to Subtract Integers

You will need
- coloured counters

GOAL
Subtract integers using a model.



Learn about the Math

Romona and Fawn were trying to calculate the difference $(-5) - (+2)$.

Fawn tried to use counters.

(●●●●●) − (●●)

She got stuck. "I have no positive counters in the first group. I can't subtract the two positive numbers in the second group from anything."

? How can you model subtraction using counters?

A. Romona suggested using blue counters. They tried $(-5) - (-2)$. Use counters to model this subtraction. What is the result?

B. Explain why $(-5) - (-2)$ is easier to model with counters than $(-5) - (+2)$.

C. Romona and Fawn went back to the original question: $(-5) - (+2)$. Fawn then remembered the zero principle, which states that $(+1) + (-1) = 0$. She said, "If we put a blue counter and a red counter together as a pair, the value is 0. If we use enough of these pairs, we will be able to do the subtraction."

●●

What is the smallest number of blue/red pairs the girls can add in order to take away two red counters?

D. What is the difference $(-5) - (+2)$?

Reflecting

1. Describe how using counters to model $(-5) - (-2)$ is different from using counters to model $(-5) - (+2)$.

2. Zeros did not need to be added to model $(-5) - (-2)$, but they were needed to model $(-5) - (+2)$.

a) Write a subtraction question with integers that requires adding zeros. Why is adding zeros required?

b) Write a subtraction question with integers that does not require adding zeros. Why is adding zeros not required?

3. Think about using counters to model a difference of two integers.

a) Describe when adding zeros is required.

b) Describe when adding zeros is not required.

4. When zeros are added during an integer subtraction, they are always added to the first term. Why?

Work with the Math

Example 1: Subtracting integers with the same sign

Calculate $(-2) - (-5)$ using a counter model.

Fawn's Solution

(●●) − (●●●●●)
= (●● 0 0 0) − (●●●●●)
= (●●●) − (●●●●●)
= (●●●)

I added three zeros to the counters in the first parentheses, so there are enough (−1) counters to subtract.

Three positive counters are left. The answer is (+3).

Example 2: Subtracting integers with opposite signs

Calculate $(+20) - (-10)$ using a counter model.

Romona's Solution

20 counters 10 counters
(● . . . ●) − (● . . . ●)

I didn't have enough counters to model this difference, so I had to imagine what it would look like.

20 counters 10 zeros 10 counters
= (● . . . ● 0 0 . . . 0 0) − (● . . . ●)
= (● . . . ● ⊖ ⊖ . . . ⊖ ⊖) − (⊖ . . . ⊖)

There were no negatives in the first group. I had to add 10 zeros to subtract the 10 negatives.

After subtracting the 10 negatives from each group, there were 30 positives left.

30 counters
= (● . . . ●)

The answer is (+30).

A Checking

5. Write the subtraction question represented by each model.

a) (●●●●) − (●●)

b) (●●●) − (●●)

c) (●●●) − (●●)

d) (●●●) − (●●)

e) (●●) − (●●●)

6. a) Identify the differences in question 5 that require zeros to be added to the first term before the subtraction can be modelled.

b) Calculate the differences in question 5.

B Practising

7. Use a counter model to calculate $(-4) - (-1)$. Why do you not need to add zeros to the model for this subtraction?

8. Use a counter model to calculate $(-1) - (-4)$. Why do you need to add zeros for this subtraction?

9. Use a counter model to show that $(-3) - (+4) = (-3) + (-4)$.

10. Use counters to explain why $(+6) - (-4)$ and $(-4) - (+6)$ are not equal.

11. Draw a counter model for each difference. Calculate each difference.

a) $(+4) - (+3)$ **e)** $(+3) - (-4)$

b) $(+3) - (+4)$ **f)** $(-4) - (+3)$

c) $(-4) - (-3)$ **g)** $(+4) - (-3)$

d) $(-3) - (-4)$ **h)** $(-3) - (+4)$

12. Draw a counter model for each difference. Calculate each difference.

a) $(+6) - (+2)$ **e)** $(+2) - (-6)$

b) $(+2) - (+6)$ **f)** $(-6) - (+2)$

c) $(-6) - (-2)$ **g)** $(+6) - (-2)$

d) $(-2) - (-6)$ **h)** $(-2) - (+6)$

13. Which subtraction statements have the same result? Use counters to explain why.

a) $(+9) - (+5)$ **d)** $(+2) - (-3)$

b) $(-4) - (-9)$ **e)** $(-4) - (+1)$

c) $(+7) - (+2)$ **f)** $(-3) - (-2)$

14. Complete each pattern.

a) $(+5) - (+4) = (+1)$
$(+5) - (+3) =$
$(+5) - (+2) =$
$(+5) - (+1) =$

b) $(-5) - (+4) = (-9)$
$(-5) - (+3) =$
$(-5) - (+2) =$
$(-5) - (+1) =$

c) $(-5) - (-9) = (+4)$
$(-5) - (-8) =$
$(-5) - (-7) =$
$(-5) - (-6) =$

d) $(-1) - (-2) = (+1)$
$(-1) - (-3) =$
$(-1) - (-4) =$
$(-1) - (-5) =$

15. Rosa used counters to represent $(-4) - (+6)$, as shown below.

Step 1 Step 2 Step 3

a) Was Rosa correct?

b) Draw your own counters to show how you would solve this question.

c) Describe each step.

16. Which differences require zeros to be added to the first term to complete the subtraction?

a) $(+100) - (+10)$ **e)** $(+10) - (-100)$

b) $(+10) - (+100)$ **f)** $(-100) - (+10)$

c) $(-100) - (-10)$ **g)** $(+100) - (-10)$

d) $(-10) - (-100)$ **h)** $(-10) - (+100)$

17. Copy and complete the following table. Use counters to model each difference. Part (a) is done for you.

	Day 1 (°C)	Day 2 (°C)	Difference
a)	−3	−1	(−1) − (−3) = (+2)
b)	+5	−4	
c)	−10	+6	
d)	−10	0	
e)	+10	+7	
f)	−8	−1	
g)	0	−10	
h)	−21	+20	

18. A golf tournament lasted two days. The players' scores for each day are shown in the following table. Miguel and Bonnie are figuring out the missing score. Copy and complete the table.

Golfer	Day 1	Day 2	Change (Day 2 – Day 1)
Ming	−5	−1	
Kaitlyn	+5	+10	
Omar	−10	+6	
Anthony	−10		−8
Braydon		−5	+10
Tynessa		+7	+2
Rana	−7		−2

19. Is each statement true or false? If a statement is sometimes true or never true, give an example to support your answer.

a) The difference between two negative numbers is always negative.

b) The difference between two positive numbers is always positive.

c) When you subtract a negative number from a positive number, the difference is always positive.

d) When you subtract a positive number from a negative number, the difference is always positive.

e) The difference between two integers always has the sign of the greater number.

20. -3 is the opposite of $+3$.

a) Why does $(-2) - (+3) = (-2) + (-3)$?

b) Is subtracting an integer always the same as adding its opposite? Use counters to explain your answer.

C Extending

21. Copy and complete the following table.

	a	b	$a - b$
a)	−215	+20	
b)	−150		−100
c)		+200	+150

22. Calculate.

a) $(+4) + (+2) - (+3)$

b) $(-4) + (-3) - (-2)$

c) $(+3) - (-8) + (-10)$

23. a) Write three different integers that make the following equation true.

$\square + \square - \square = (-10)$

b) Find two other solutions.