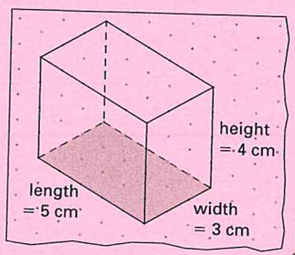
11.2 Volume of a Rectangular Prism

You will need
- centimetre grid paper
- centimetre linking cubes
- a calculator
- a ruler
- triangle dot paper

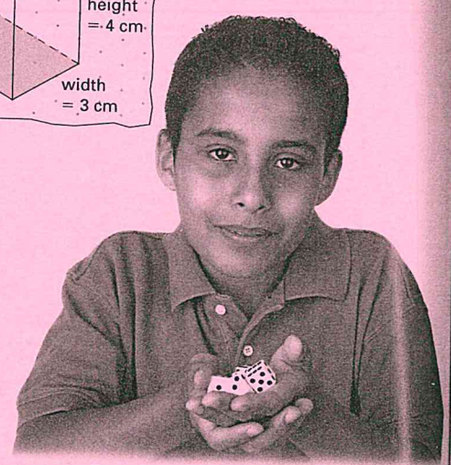
▶ **GOAL**
Develop a formula to calculate the volume of a rectangular prism.

Learn about the Math

James says, "I have to pack dice in boxes like this. Each edge of each die is 1 cm long. If I calculate the volume of a box, I'll know how many dice it can hold."



? How many centimetre linking cubes can you put in the box?



A. Model James's problem. Use centimetre grid paper to represent the base of the box. Use centimetre linking cubes to represent the dice. What is the area of the base?

B. Stack the cubes to the height of the box. How many layers of cubes do you have?

C. Use your answers in steps A and B to calculate the volume of the box.

D. Repeat steps A to C using a different face as the base of the box.

Reflecting

1. How many choices do you have when selecting a face to use as the base of the box?

2. How does your choice for the base affect the height of the box?

3. How does your choice for the base affect your calculation of the volume of the box?

4. Write a formula that describes how to calculate the volume of a rectangular box, no matter which face you choose as the base.

Communication Tip

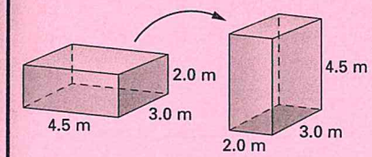
Units of volume always have a small, raised 3 written after them: $12\ m^3$. The raised 3 indicates that three dimensions are involved: length, width, and height.

Work with the Math

Example 1: Calculating volume when the sides are not all whole numbers

Calculate the volume of a rectangular box that measures 3.0 m by 4.5 m by 2.0 m.

Indira's Solution



First I sketched the box. To make the calculations easier, I used the face that is 2.0 m by 3.0 m as the base.

Area of base = length × width
= 2.0 m × 3.0 m
= $6.0\ m^2$

I multiplied the length by the width to find the area of the base.

Volume = Area of base × height
= $6.0\ m^2 \times 4.5\ m$
= $27.0\ m^3$

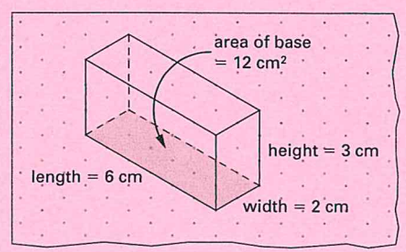
I multiplied the area of the base by the height to find the volume.

The volume of the box is $27.0\ m^3$.

Example 2: Determining dimensions of a box with a given volume

What are some possible dimensions of a rectangular box with a volume of $36\ cm^3$?

Simon's Solution



I modelled the box using 36 centimetre linking cubes.

I put 12 centimetre linking cubes in the first layer.

I stacked two more layers to complete the model.

Then I drew my model on triangle dot paper.

Volume = length × width × height
$36\ cm^3$ = 6 cm × 2 cm × 3 cm
$36\ cm^3$ = 4 cm × 3 cm × 3 cm

I wrote the dimensions for my diagram. Then I calculated a different way to form a base of $12\ cm^2$. I think there are others. What are they?

A Checking

5. Which coloured face would you use for the base if you wanted to calculate the volume of each prism? Why?

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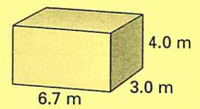

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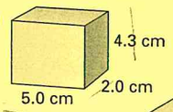

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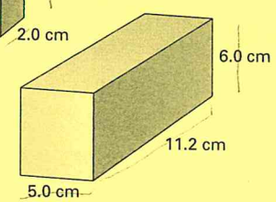
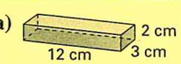


6. Calculate the volume of each prism in question 5.

B Practising

7. Calculate the volume of each prism.

a)



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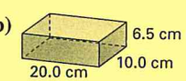

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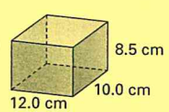

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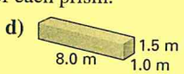

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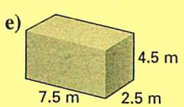

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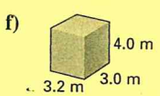


8. a) Sketch a rectangular prism that has a volume of 500 cm^3 and sides that are whole numbers.

b) Sketch a rectangular prism that has a volume of 500 cm^3 and some sides that are *not* whole numbers.

9. Sketch a rectangular prism with each set of dimensions. Calculate the volume of the prism.

a) $l = 8$ cm, $w = 8$ cm, $h = 8$ cm

b) $l = 0.5$ m, $w = 0.5$ m, $h = 2.0$ m

c) $l = 3.5$ km, $w = 2.0$ km, $h = 3.0$ km

10. Which rectangular prism in the table below would you use to pack each item? Choose each prism only once. Explain your choices.

A. a box of pencils

B. a basketball

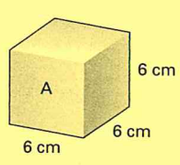
C. earrings

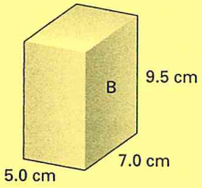
D. one CD

E. ten CDs

	Length (cm)	Width (cm)	Height (cm)
a)	14	12	1
b)	6	6	1
c)	40	40	40
d)	14	12	10
e)	19.0	3.5	1.5

11. Anthony needs to buy a box of nails for his carpentry project. The hardware store sells these two boxes of nails for the same price. Which box should Anthony buy? Explain your choice with a sketch, calculations, and words.





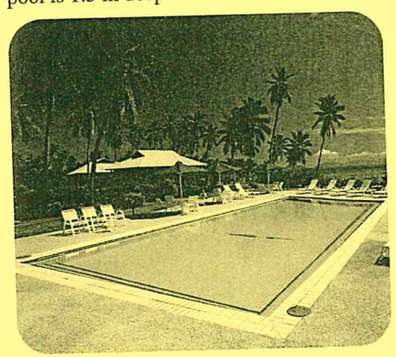
12. List two sets of dimensions (length, width, and height) that will result in a box with each volume.

a) 24 cm^3 b) 27 cm^3 c) 51.2 cm^3

13. Copy and complete the table by filling in the missing dimensions of the rectangular prisms.

	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a)	6	6	8	
b)	4.5	5.0		216.0
c)	3		3	27
d)	2.5	8.4		52.5
e)		7	7	343

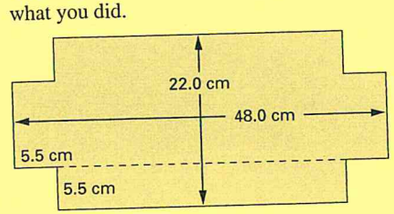
14. This pool is 10 m long by 6 m wide. The pool is 1.5 m deep.



a) If a volume of 1 m^3 holds 1000 L, how many litres of water can the pool hold?

b) How many litres of water are in the pool when the pool is 90% full?

15. Regan folded this net into an open box. Calculate the volume of the box. Explain what you did.



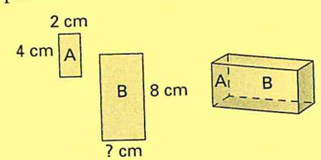
16. Brand A tissues come in a box that is 22.5 cm long, 12.0 cm wide, and 6.8 cm high. It sells for 79¢. Brand B tissues come in a box that is 13 cm long, 13 cm wide, and 8 cm high. It sells for $1.99. Which brand is the better buy? Explain your answer.

17. Humidifier W is 37.0 cm long, 24.0 cm wide, and 23.5 cm high. Humidifier X is 50 cm long, 40 cm wide, and 11 cm high. Humidifier Y is 40 cm long, 35 cm wide, and 20 cm high. Which humidifier holds the most water? Explain your answer.

C Extending

18. In each case, rectangle A has the same shape as rectangle B, but it is a different size. The rectangles form the sides shown in each prism. Determine the unknown side length of B, and calculate the volume of each prism.

a)



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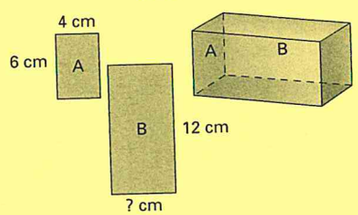

19. A calculator is tightly wrapped in protective padding. It is sold in a box that is 14.5 cm wide, 24.0 cm long, and 6.0 cm high. The calculator itself is 8.5 cm wide, 18 cm long, and 2.5 cm high. What is the volume of the padding?