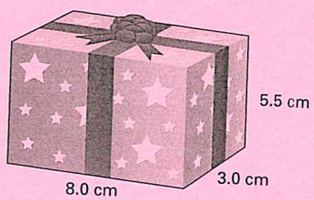
11.1 Surface Area of a Rectangular Prism

You will need
- a calculator
- centimetre square dot paper
- triangle dot paper
- a ruler

▶ **GOAL**
Develop a formula to calculate the surface area of a rectangular prism.

Learn about the Math

Kwami and Jody are wrapping small boxes of wedding cake for souvenirs at a wedding. Kwami says, "We'll need enough wrapping paper to cover the surface, plus about 10% to allow for overlaps and folding."

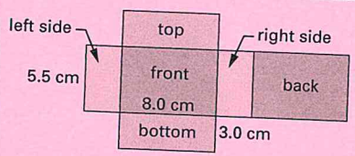


? How much paper do Kwami and Jody need to wrap each box?

Example 1: Determining surface area by adding the areas

Use a net to determine the amount of wrapping paper that Kwami and Jody need to wrap each box.

Kwami's Solution



I imagined unwrapping the box and laying the paper flat. I drew the net of the box, and labelled all the faces.

Each face is a rectangle.

I calculated the area of each face using the formula $A = l \times w$.

Area of front $= 8.0\text{ cm} \times 5.5\text{ cm}$
$= 44.0\text{ cm}^2$

Area of back $= 8.0\text{ cm} \times 5.5\text{ cm}$
$= 44.0\text{ cm}^2$

Area of right side $= 3.0\text{ cm} \times 5.5\text{ cm}$
$= 16.5\text{ cm}^2$

Area of left side $= 3.0\text{ cm} \times 5.5\text{ cm}$
$= 16.5\text{ cm}^2$

Area of top $= 8.0\text{ cm} \times 3.0\text{ cm}$
$= 24.0\text{ cm}^2$

Area of bottom $= 8.0\text{ cm} \times 3.0\text{ cm}$
$= 24.0\text{ cm}^2$

To find the total surface area, I added these areas.

$$\begin{aligned}\text{Total Surface Area} &= \text{front} + \text{back} + \text{right side} + \text{left side} + \text{top} + \text{bottom}\\ &= 44.0\text{ cm}^2 + 44.0\text{ cm}^2 + 16.5\text{ cm}^2 + 16.5\text{ cm}^2 + 24.0\text{ cm}^2 + 24.0\text{ cm}^2\\ &= 169.0\text{ cm}^2\end{aligned}$$

The total surface area is 169.0 cm^2.

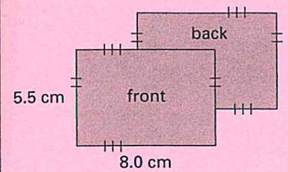
10% of 169.0 cm^2 is 16.9 cm^2.

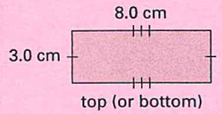
The total amount of wrapping paper that we need is $169.0\text{ cm}^2 + 16.9\text{ cm}^2 = 185.9\text{ cm}^2$.

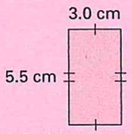
Example 2: Determining surface area by doubling congruent areas

Determine the amount of wrapping paper that Kwami and Jody need to wrap each box.

Jody's Solution







I noticed that the front face and the back face have the same area. Also, the top face and the bottom face have the same area. The same is true for the left side and the right side.

I just need to find the area of one of each rectangle, and double it.

$$\begin{aligned}\text{Area of front and back} &= 2 \times (8.0\text{ cm} \times 5.5\text{ cm})\\ &= 2 \times 44.0\text{ cm}^2\\ &= 88.0\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of top and bottom} &= 2 \times (8.0\text{ cm} \times 3.0\text{ cm})\\ &= 2 \times 24.0\text{ cm}^2\\ &= 48.0\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of both sides} &= 2 \times (5.5\text{ cm} \times 3.0\text{ cm})\\ &= 2 \times 16.5\text{ cm}^2\\ &= 33.0\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total Surface Area} &= 88.0\text{ cm}^2 + 48.0\text{ cm}^2 + 33.0\text{ cm}^2\\ &= 169.0\text{ cm}^2\end{aligned}$$

I added 10% more to allow for overlaps and folding.

The total is 185.9 cm^2, but I think I should round it so the answer is reasonable.

We need about 190 cm^2 of wrapping paper for each box.

$$\begin{array}{r} 16.9\text{ cm}^2 \\ +\ 169.0\text{ cm}^2 \\ \hline 185.9\text{ cm}^2 \end{array}$$

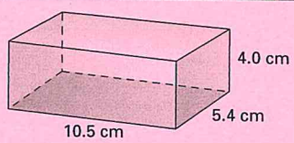
Reflecting

1. How are Kwami's and Jody's solutions alike? How are they different?
2. Why was Jody able to do half the area calculations that Kwami had to do?
3. After Kwami was done, he noticed that he could add the area of the front, one side, and the top, and then double the sum to get the same answer. Check if he is correct.
4. Write a formula to calculate the surface area of a rectangular prism.

Work with the Math

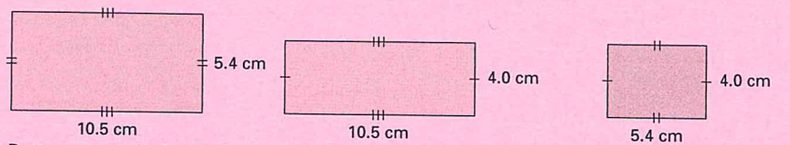
Example 3: Calculating surface area using a formula

Calculate the surface area of this rectangular box.



Solution A

Sketch the different faces.



Bottom or top is the 1st face. Front or back is the 2nd face. Either side is the 3rd face.

Surface Area = (2 × Area of 1st face) + (2 × Area of 2nd face) + (2 × Area of 3rd face)
= (2 × 10.5 cm × 5.4 cm) + (2 × 10.5 cm × 4.0 cm) + (2 × 5.4 cm × 4.0 cm)
= 113.4 cm^2 + 84.0 cm^2 + 43.2 cm^2
= 240.6 cm^2

Solution B

length = 10.5 cm, width = 5.4 cm, height = 4.0 cm

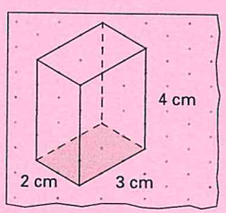
Area of bottom or top	Area of front or back	Area of either side
= length × width	= length × height	= width × height
= 10.5 cm × 5.4 cm	= 10.5 cm × 4.0 cm	= 5.4 cm × 4.0 cm
= 56.7 cm^2	= 42.0 cm^2	= 21.6 cm^2

Surface Area = 2 × (sum of the 3 different faces)
= 2 × (56.7 cm^2 + 42.0 cm^2 + 21.6 cm^2)
= 2 × 120.3 cm^2
= 240.6 cm^2

A Checking

5. a) Draw a net for this rectangular prism on centimetre square dot paper. Then calculate the surface area.

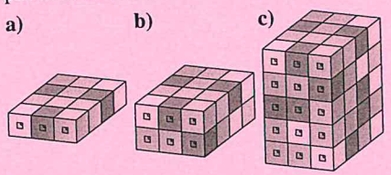
b) Use a formula to calculate the surface area of the prism.



B Practising

6. State the length, width, and height of each prism. Then calculate each surface area.

a) **b)** **c)**

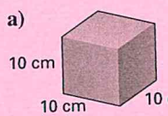


7. a) Sketch a rectangular prism that is 3 cm by 5 cm by 6 cm.

b) What is the surface area?

8. Determine the surface area of each cube.

a)



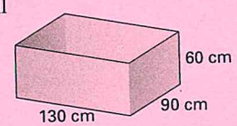
b)

9. a) List the dimensions of all the rectangular prisms you could make with twelve centimetre linking cubes.

b) Which prism has the greatest surface area?

c) Which prism has the least surface area?

10. Julie built a small scene inside an open box for a history project. She is wrapping the box to take it to school.



a) Exactly how much wrapping paper is needed to cover the outside of the box, including the top?

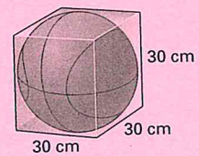
b) How much wrapping paper will Julie need if she needs an extra 10% for overlaps and folds?

11. Karen is painting the floor and four walls of a basement. The basement is 12.0 m long, 4.0 m wide, and 2.5 m high. Paint comes in 4 L cans. One litre of paint covers 10 m^2.

a) What area does Karen need to paint?

b) How many cans of paint does Karen need?

12. A sports company packages its basketballs in boxes, as shown. The boxes are shipped in wooden crates. Each crate holds 24 boxes.



a) Sketch all the possible shapes of crates that could be used. The dimensions must be whole numbers of centimetres.

b) Calculate the surface area of each crate you sketched in part (a).

c) Which crate is made from the least amount of wood?

13. Jordan cuts a cube into small pieces. Is the total surface area of all the pieces less than, greater than, or equal to the surface area of the original cube? Explain your thinking with words, diagrams, and calculations.

C Extending

14. a) Sketch any rectangular prism that has a surface area of 24 cm^2.

b) Double the lengths of the sides to draw a new rectangular prism. How does the surface area of your new prism compare with the surface area of your original prism?

c) Sketch another new rectangular prism with sides that are half the length of the original prism's sides. How does the surface area of this new prism compare with the surface area of your original prism?