

# 6.3

## Adding Integers Using the Zero Principle

You will need  
• red and blue counters

**GOAL**

Use the zero principle, with and without models, to add integers.

### Learn about the Math

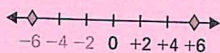
In a coin tossing experiment, Paul gained 1 point (+1) when he tossed Heads. He lost 1 point (-1) when he tossed Tails.

The following table shows Paul's results of 11 tosses.

Toss number	1	2	3	4	5	6	7	8	9	10	11
Result (+1) or (-1)	-1	-1	+1	+1	+1	-1	+1	-1	+1	-1	-1

**opposite integers**

two integers the same distance away from zero; for example, +6 and -6 are opposite integers



**zero principle**

two opposite integers, when added, give a sum of zero; for example, (-1) + (+1) = 0

? How can you add integers to calculate Paul's score after 11 tosses?

You can use a blue counter ● to represent (-1) and a red counter ● to represent (+1). The integers (+1) and (-1) are **opposite integers**.

Adding (-1) and (+1) gives a net result of zero.

This is the **zero principle**.

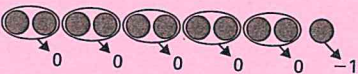
#### Example 1: Modelling the sum with counters

Use counters to calculate Paul's score after 11 tosses.

**Paul's Solution**



I modelled my first 11 tosses using counters. I used blue counters to represent (-1) and red counters to represent (+1).



I changed the order to get pairs of blue and red counters. In each pair, (+1) paired with (-1) is 0. One blue counter was left over. The answer is (-1).



#### Example 2: Using +1s and -1s

Use positive 1s and negative 1s to calculate Paul's score after 11 tosses.

**Fawn's Solution**

$$(-1) + (-1) + (+1) + (+1) + (+1) + (-1) + (+1) + (-1) + (+1) + (-1) + (-1) = (-1)$$

I found pairs of positive 1s and negative 1s.

I added the +1 and the -1 in each pair to get 0.

There was a single -1 left over.



#### Example 3: Combining integers greater than +1 and less than -1

Use integers greater than +1 and less than -1 to calculate Paul's score after 11 tosses.

**Miguel's Solution**

$$(+5) + (+5) + (-5) + (-5) + (-1) = (-1)$$

I wrote all the positive 1s first. Then I wrote all the negative 1s.

$$\begin{aligned} & (+5) + (-5) + (-1) \\ &= 0 + (-1) \\ &= (-1) \end{aligned}$$

The zero principle says that (-5) + (+5) = 0. (-1) was left over.



### Reflecting

- a) How are Paul's and Fawn's solutions alike?  
b) How are they different?
- Miguel used the idea that the sum of any two opposite integers is always zero. Verify Miguel's solution using counters.

### Communication Tip

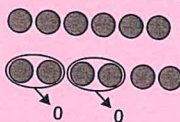
- Read the integer +1 as "positive 1" and the integer -1 as "negative 1."
- Integer expressions often have brackets around the integers.
- Do not confuse the sign of an integer with the operation of addition or subtraction. For example, to add +2 and -4, write (+2) + (-4). This means "positive 2 plus negative 4."

## Work with the Math

### Example 4: Using the zero principle

Add  $(-4) + (+2)$ .

#### Solution A: Using counters



The answer is  $(-2)$ .

#### Solution B: Using numbers greater than +1 and less than -1

$$\begin{aligned} (-4) + (+2) &= (-2) + (-2) + (+2) \\ &= (-2) + 0 \\ &= -2 \end{aligned}$$

The answer is  $(-2)$ .

### A Checking

3. Add each expression using counters and numbers.

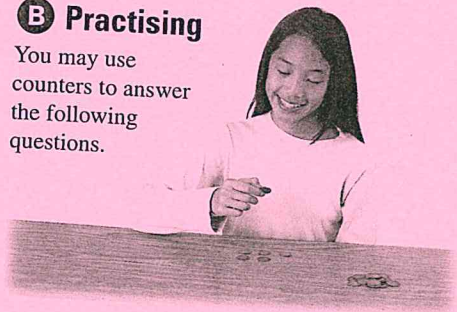
	Expression	Using counters	Using numbers
a)	$(-3) + (+2)$		
b)	$(-4) + (+6)$		
c)	$(+5) + (-6)$		
d)	$(-5) + (+7)$		
e)	$(+2) + (-8)$		
f)	$(-1) + (-9)$		

4. Use mental math to determine each sum.

- a)  $(+3) + (-3) =$   
 b)  $(-7) + (+7) =$

### B Practising

You may use counters to answer the following questions.



5. Complete.

- a)  $(-3) + (-2) =$   
 b)  $(+2) + (-2) =$   
 c)  $(-4) + (+1) =$   
 d)  $(-7) + (+6) =$   
 e)  $(-5) + (-2) =$   
 f)  $(-5) + (+2) =$

6. Explain why  $(-25) + (+25) = 0$ .

7. The following patterns are based on adding integers. Continue each pattern. Then write a rule to describe each pattern.

- a)  $0, -1, -2, -3, -4, \dots$   
 b)  $-3, -2, -1, 0, \dots$

8. Fill in each with  $+1$  or  $-1$  to make each statement true.

- a)  $(+1) + \quad + \quad = (-1)$   
 b)  $(-1) + \quad + \quad = (+1)$   
 c)  $(+1) + \quad + \quad + \quad + \quad = (-1)$   
 d)  $(+1) + \quad + \quad + \quad + (+1) = (-1)$

9. Complete.

- a)  $(-3) + (+3) + (+5) =$   
 b)  $(-1) + (-2) + (-1) =$   
 c)  $(+2) + (+1) + \quad = (-1)$

10. Use  $=$ ,  $<$ , or  $>$  to make each statement true.

- a)  $(-1) + (-2) \quad (-4)$   
 b)  $(+2) + (-5) \quad (-3)$   
 c)  $(-3) + (+6) \quad (+2)$   
 d)  $(+5) + (-7) \quad (-2)$   
 e)  $(-2) + (-4) \quad (-5)$   
 f)  $(-2) + (+1) \quad 0$

11. Using  $+1$  and  $-1$  only, create an addition question that has each sum. Use at least four numbers for the question. Check your work using counters.

- a)  $+3$     b)  $-2$     c)  $0$     d)  $-1$

12. a) Calculate the sum. You can use counters or numbers.

$$\begin{aligned} (+1) + (+1) + (-1) + (+1) + (-1) + \\ (-1) + (+1) + (+1) + (-1) \end{aligned}$$

b) Which method did you choose? Why?

13. Fill in each with an integer to make the equation true. Show three different solutions.

$$\quad + \quad + \quad = (-5)$$

14. Explain why you cannot complete this equation using only  $+1$ s or  $-1$ s.

$$(+1) + \quad + \quad + \quad = (+1)$$

15. In a Magic Square, all rows, columns, and diagonals have the same sum. No number appears more than once.

a) This Magic Square uses integers from  $-6$  to  $+2$ . Verify that the rows, columns, and diagonals all have the same sum.

$+1$	$-6$	$-1$
$-4$	$-2$	$0$
$-3$	$+2$	$-5$

The sum of the  $(-1) + 0 + (-5) = (-6)$  third column is shown.

b) This Magic Square uses integers from  $-1$  to  $+7$ . Complete it. Check that all the sums are the same.

$+2$		
$+7$	$+3$	$-1$

c) This Magic Square uses integers from  $-4$  to  $+4$ . Complete it. Check that all the sums are the same.

$+3$	$-4$	
$-2$		
	$+4$	

d) Create a Magic Square that uses integers from  $-10$  to  $-2$ .

### C Extending

16. State whether each statement is true or false. Explain your reasoning.

- a) The sum of two positive integers is positive.  
 b) The sum of two negative integers is negative.  
 c) The sum of a negative integer and a positive integer is always positive.

17. Continue each pattern. Write a rule to describe the pattern.

- a)  $0, +1, -1, +2, -2, +3, -3, \dots$   
 b)  $-1, 0, -1, -1, -2, -3, -5, -8, \dots$

18. Without using a calculator, determine the sum of all the integers from  $-50$  to  $+50$ . Describe your strategy.

19. a) Calculate the average daily high temperature for the four days.

Four-day weather forecast	High temperature ( $^{\circ}\text{C}$ )	Low temperature ( $^{\circ}\text{C}$ )
Wednesday	$+10$	$0$
Thursday	$+5$	$-6$
Friday	$+9$	$-7$
Saturday	$+8$	$-3$

- b) Calculate the average daily low temperature for the four days.  
 c) What is the range between the highest high and the lowest low?